Now if $\theta=\alpha$, the plane then being parallel to one and only one element, (5) reduces to $y^2-4gx\sin^2\alpha=0$, a parabola of latus rectum $=4g\sin^2\alpha$.

If $\theta>\alpha$ the section cuts all elements and the coefficients of x^2 and y^2 are both positive, and we have an ellipse whose center, axes, and eccentricity are readily found; and in particular if $\theta=90°$, the section is parallel to the base, the coefficients of x^2 and y^2 are unity and we have a circle, whose center is $(g\sin\alpha, 0)$. If $\theta<\alpha$, the section cuts both nappes, the coefficients of x^2 and y^2 are of unlike sign and we have a hyperbola.

If $g=0$, (5) becomes $y=\pm x\sqrt{\dfrac{\sin^2\alpha-\sin^2\theta}{\cos^2\alpha}}$....(6). Now if $\theta=\alpha$, (6) becomes $y=0$, a straight line, the limit of the parabola. If $\theta<\alpha$, (6) represents two real straight lines, the limiting case of the hyperbola. And if $\theta>\alpha$, (6) represents two imaginary lines intersecting in the real point (0, 0), which is the limiting form of the ellipse.

The equations (5) and (6) show the dependence of the nature of the conic sections upon the angle which the cutting plane makes with the axis, and the dependence of their shape upon the angle of the cone and the distance from the vertex to the first element cut.

Remark 1. If the section be a parabola, the foot of the perpendicular from the middle point of the line through D parallel to BC, upon DH, is the focus.

Remark 2. The eccentricity of any conic section is $\varepsilon=[\cos\theta/\cos\alpha]$.

NEW AND OLD PROOFS OF THE PYTHAGOREAN THEOREM.

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[Continued from November Number.]

LXX. Fig. 31.
$AEM=ACB$ of $AEHC$.
$MOL=ACB=ADK+DKBC=BHI+DKBC$. $LOI=BEK$.
$\therefore ABLM \backsimeq BCDF+AEHC$.

LXXI. Fig. 31.
$AMPQ \backsimeq AMOC \backsimeq AEHC$. $BLPQ \backsimeq BLOC \backsimeq BCDF$.
$\therefore ABLM \backsimeq BCDF+AEHC$.

LXXII. Fig. 31.
$MTE=BSF$. $\therefore BS=MT$. $\therefore AMLB \backsimeq 2AMTS$.
But $AMTS \backsimeq EAC+FBC$; since $MTE=BSF$, and $AEM=ACB$.
$\therefore 2AMTS \backsimeq 2EAC+2FBC \backsimeq AEHC+BCDF$.
$\therefore AMLB \backsimeq BCDF+AEHC$.

LXXIII. Fig. 31.

$ABVU \simeq ABHW$. $UVLM \simeq 2MLH \simeq 2MOL + 2LOH \simeq HEW + ACB + 2FBC (\simeq BCDF)$.

$\therefore ABLM \simeq BCDF + AEHC$.

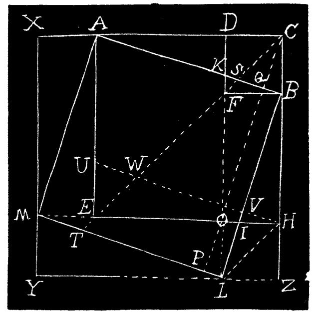

Fig. 31.

LXXIV. Fig. 31.

$ABLM \simeq CZYX - 4ACB$.

$BCDF + AEHC = OLZH + AEHC \simeq CZYX - 2MOLY \simeq CZYX - 4ACB$.

$\therefore ABLM \simeq BCDF + AEHC$.

Note.—This proof is similar to that of Henry Boad's, London, 1733.

LXXV. Fig. 32.

$ANML \simeq 2ACL = 2AFB \simeq AFHC$.

$NMKB \simeq 2BCK \simeq BCDE$.

$\therefore ABKL \simeq BCDE + AFHC$. *Wipper.*

LXXVI. Fig. 32.

Since EC produced to O passes through the center of $ABKL$, $ABQO \simeq \frac{1}{2}ABKL$. Now, $APCO \simeq CAF$, since $ACO = AFP$; $PBQC \simeq CBE$, since $BCP = BEQ$. $\therefore ABKL \simeq BCDE + AFHC$.

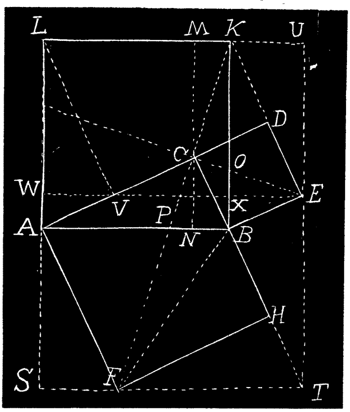

Fig. 32.

LXXVII. Fig. 32.

$LVDK = AFHB$. Then, $AVL = BEK$.

$\therefore ABKL \simeq BCDE + AFHC$.

LXXVIII. Fig. 32.

$ABKL \simeq STUL - 2ASF - 3FHT - AFHB$.

$BCDE + AFHC \simeq STUL - 2ASF - 3FHT - LVDK (= AFHB)$.

$\therefore ABKL \simeq BCDE + AFHC$.

LXXIX. Fig. 32.

$ABXW \simeq ABEV \simeq BCDE$.

$WXKL \simeq VEKL \simeq AFHC$, since $VED = ABC$, and $VDKL = FHBA$.

$\therefore ABKL \simeq BCDE + AFHC$.

LXXX. Fig. 33.

$AOML \simeq ANCL \simeq 2ANC \simeq AFHC$.

$OBKM \simeq 2BCK \simeq BCDE$, since SK, the altitude of BCK, $= BC$.

$\therefore ABKL \simeq BCDE + AFHC$.

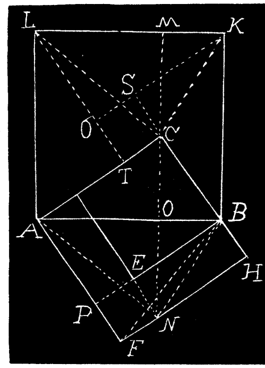

Fig. 33.

LXXXI. Fig. 33.

$AOML \simeq 2ACL = 2AFB \simeq AFHC$.

$OBKM \simeq BKCN \simeq 2BCN \simeq BCDE$.

$\therefore ABKL \simeq BCDE + AFHC$.

LXXXII. Fig. 33.

$QTCS = PFNE$.

$ATL + LQK + KSB = APB + CDN + NHC$.

$\therefore ABKL \simeq BCDE + AFHC$.

[To be Continued.]